Capturing Fuzziness and Uncertainty of Spatiotemporal Objects

Dieter Pfoser and Nectaria Tryfona

Computer Science Department, Aalborg University
Fredrik Bajersvej 7E, DK-9220 Aalborg East, Denmark
{pfoser,tryfona}@cs.auc.dk

Abstract. For the majority of spatiotemporal applications, we assume that the modeled world is precise and bound. This simplification seems unnecessary crude for many environments handling spatial and temporal extents, such as navigational applications. In this work, we explore fuzziness and uncertainty, which we subsume under the term *indeterminacy*, in the *spatiotemporal* context. We first show how the fundamental modeling concepts of *spatial objects*, *attributes*, *relationships*, *time points*, *time periods*, and *events* are influenced by *indeterminacy*, and then show how these concepts can be combined. Next, we focus on the *change* of spatial objects according to their geometry over time. We outline four scenarios, which identify discrete and continuous change, and we present how to model indeterminate change. We demonstrate the applicability of this proposal by describing the uncertainty related to the movement of point objects, such as the recording of the whereabouts of taxis.

1 Introduction

Spatiotemporal applications received a lot of attention over the past years. Requirements analysis [15], models [4], data types [8], and data structures [14] are some of the main topics in this area. Although considerable research effort and valuable results *do* exist, all the studies and approaches are based on the assumption that, in the spatiotemporal mini-world, objects have *crisp* boundaries, relationships among them are *precisely* defined, and *accurate* measurements of positions lead to error-free representations.

However, reality is different. Very often boundaries do not strictly separate objects but, rather, show a transition between them. Consider the example from an environmental system in which the different soil zones, such as desert and prairie, are not precisely bound. We encounter a transition, or *fuzziness*, between them. On the other hand, in navigational systems, the position of a moving vehicle, although precise in its nature, might not be exactly known, e.g., car A is in New York. We encounter *uncertainty,* i.e., *lack of knowledge* or *error* about its actual location.

In this paper, we deal with *fuzziness and uncertainty as related to spatiotemporal objects*. More specifically, we start by pointing out the semantic differences between the two cases that constitute *spatiotemporal indeterminacy*: *fuzziness,* concerning “blurry” situations, and *uncertainty,* expressing the “not-exactly-known” reality. We

A. Caplinskas and J. Eder (Eds.): ADBIS 2001, LNCS 2151, pp. 112-126, 2001.

clarify these terms in the spatial and temporal domains, as well as the combined effect, i.e., spatiotemporal fuzziness and uncertainty. We show how the basic spatiotemporal modeling concepts of spatial objects, attributes, relationships, time points, time periods, events, and change are influenced by indeterminacy. We provide formal ways to describe this, while an example demonstrates the applicability of this proposal. A more elaborate discussion with the use of fuzzy set and probability theory in this area can be found in [16].

There are only few works towards spatiotemporal indeterminacy. [18] focuses on simple, abstract, spatial and temporal uncertainty concepts and integrates them to describe spatial updates in a GIS database. [13] discusses spatiotemporal indeterminacy for moving objects data. It is, however, limited to point objects and it does not take temporal errors into account. [2] aims at describing the change of fuzzy features over time using a raster representation. More work exists towards temporal, e.g., [5] and spatial indeterminacy, e.g., [1], [3], [7], [17], [19], [20].

The rest of the paper is organized as follows. Section 2 briefly presents the fundamental spatial and temporal concepts involved in the spatiotemporal application domain. Section 3 explores the semantics and gives the mathematical expression of indeterminate temporal concepts. Section 4 deals with indeterminate spatial concepts. Section 5 discusses change as the spatiotemporal concept affected by indeterminacy. Finally, Sect. 6 concludes with the future research plans.

2 Spatial and Temporal Concepts

To understand spatiotemporal indeterminacy, it is important to realize the fundamental spatial, temporal, and spatiotemporal concepts.

Spatiotemporal applications can be categorized based on the type of data they manage: (a) applications dealing with *moving* objects, such as navigational, e.g., a moving “car” on a road network, (b) applications involving objects located in space, whose characteristics and their position, may *change* in time, e.g., in a cadastral information system, “landparcels” change positions by changing shape, but they do not “move,” and (c) applications integrating the above two behaviors, e.g, in environmental applications, “pollution” is measured as a *moving* phenomenon which *changes* properties and shape over time. The following modeling concepts are involved in environments like the aforementioned.

- *Spatial Objects* and their *geometry.* Spatial objects are objects whose position in space *matters*, e.g., a moving “car.” Many times, *not only the actual* object’s *position* matters, but its *geometry* does as well. For example, in a cadastral system the *exact geometry* of a “landparcel” is of importance. The geometry of the position of a spatial object can be (of type) point, line, region or any combination thereof [10].
- *Spatial Relationships.* Spatial relationships relate spatial objects, or more precisely, the positions of the objects, e.g., two landparcels share common borders.
- *Spatial Attributes* and their *geometry.* Spatial objects have, apart from descriptive attributes, also spatial attributes, e.g., the “vegetation” of a “landparcel.” Values of spatial attributes depend on the referenced position and not on the object itself.

Spatial attributes are also related to geometries in space, as they split space in parts in whose extents the values of the spatial attributes remain the same; each part of space has (like, the objects' positions) geometry (of type) point, line, region or any combination thereof. For example, the attribute "vegetation" creates partitions of space with constant vegetation values in each partition, such as "forest", and "bushes." There are two types of spatial attributes, (a) those representing functions with continuous range, e.g., "temperature." Here the geometry of the partitions is point. (b) Those representing functions with discrete range, e.g., "vegetation" is represented as a set of regions. Not all spatial objects have spatial attributes. For example, no spatial attribute is usually assigned to a moving "car," while various ones, e.g., "soil type," may be assigned to a "landparcel."

- *Time.* Many different models of time exist. Some authors even propose taxonomies of time. In our work we assume a linear ordered time line, isomorphic to a finite subset of the natural numbers. The elements of this set are termed *chronons*.
- *Time points vs. Time periods.* Two basic models of time are used to record facts and information of a database: time point and time period. A time point t_1 is located during a chronon, while a time period $[t_k, t_m]$, with t_k, t_m time points and $k \leq m$ has a duration and is defined as a set of chronons.
- *Events* and *States.* An event occurs at an exact time point, i.e., an event has no duration. For example a "car crash.'' A state is defined for each chronon in a time point. It has duration, e.g., a "meeting" takes place from 9 am until 11am.

3 Temporal Indeterminacy

In temporal applications we are interested in events and their occurrence time. However, sometimes we only know approximately when an event occurred, e.g., a traffic accident happened between "2 pm and 4 pm." Next, we present models to represent indeterminacy in the temporal domain by adapting the model presented in [5].

3.1 Indeterminate Time Points

A time point is *determinate* if it is known during which chronon it is located. Fig 1a shows the determinate point I_1, based on the approach that a chronon is longer than a time point. A time point is *indeterminate* if it is not known exactly when, but approximately during which series of chronons it is located. An indeterminate time point is described by a *lower support*, an *upper support*, and a *probability function* [5]. The *supports* are chronons that delimit the location of the time point, e.g., for time point I_2 in Fig. 1a, the lower support is 5 and the upper support is 8; the probability function shows the likelihood where the time point is located within the range, e.g., in uniform distribution, it is equally likely for the time point to be located at chronons 5 to 8.

In the following, we use probability and fuzzy set theory to quantify indeterminacy. The probability mass function, p_x, for the indeterminate point x is

$$p_x(i) = P[x = i] : i \in \mathbb{N} \tag{1}$$

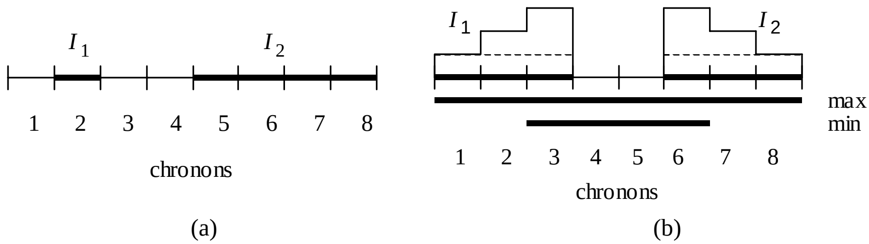


Fig. 1. (a) Determinate (I_1) and indeterminate (I_2) time points, (b) indeterminate time period, probabilities of bounding time points (solid line-probability density function, dashed line-probability mass function)

where $P[x = i]$ is the probability that the time point is located during chronon *i*. In our example, assuming uniform distribution, $P[I_2 = 6] = 0.25$, the probability outside the range lower support–upper support is 0. Also, all indeterminate time points are considered to be independent, i.e.,

$$P[x = i \wedge y = j] = P[x = i] \times P[y = j] \tag{2}$$

We can state that all probability distributions are fuzzy sets [16]. By using the probability mass function as basis we obtain the following membership function

$$\mu_x(i) = \lambda p_x(i) \tag{3}$$

where λ is an arbitrary scale factor relating the membership grade to the probability.

3.2 Indeterminate Time Periods

A time period is a subset of the time line bound by two time points. Depending on whether the bounding points are determinate or indeterminate, we term the time period accordingly. In Fig. 2b, I_1 and I_2 denote the indeterminate start and end point of the period. Possible periods can range from chronon 1 to chronon 8 (max), but at least have to range from 3 to 6 (min).

The time period presented in Fig. 1b can also be perceived as having a fuzzy boundary. Next, we derive a membership function, $\mu_T(x)$, returning the degree to which an arbitrary chronon *x* is part of the time period *T*. From Fig. 1b, we can deduce that chronons 4 and 5 are definitely part of the time period *T*, whereas other chronons might be. Assuming a uniform distribution of the chronons within the time points I_1 and I_2, we can see that if chronon 2 is within the period so has to be chronon 3. Further, if chronon 1 is within, so have to be chronons 2 and 3. The same is true for chronons 6, 7, and 8 of I_2. Thus, in three cases chronon 3, in two cases chronon 2, and in one case chronon 1 is within period *T*. The probability mass function of I_1 and I_2 as shown in Fig. 1b gives the probability for a chronon to be in *T*. In summing up the probability from "the outside to the inside," we obtain a step function, the probability density function.

To derive the membership function, $\mu_T(x)$, we have to split the time period *T* into three parts; (1) the "core" (chronons 4 and 5), (2) the intervals I_1 and I_2, and (3) the outside world. A membership grade of 1 and 0 indicate definite and no membership in the time period, respectively. All chronons in the core have a grade of 1. The grade of the chronons in the intervals is equal to the value of the probability density function. Formula 4 summarizes the membership function.

$$\mu_T(x) = \begin{cases} 1 & y \text{ in core} \\ \sum p(x) & y \in I_1 \vee I_2 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

4 Spatial Indeterminacy

In the spatial indeterminacy area, [9] states that fuzziness is a property of a geographic entity. Fuzziness concerns objects that cannot be precisely defined otherwise [6]. On the other hand, uncertainty results from limitations of the observation, i.e., the measurement process [9].

4.1 Indeterminate Spatial Objects, Relationships, and Attributes

In the following, we point out the differences between spatial fuzziness and spatial uncertainty more prominently. Consider the example of the different soil zones, e.g., desert and prairie. Each zone is not precisely bound, but, rather, a *blurry* situation exists around their common boundaries. We can identify a location for which we are sure it is within the desert or the prairie, and we can find a location that is in-between. Consequently, the boundary between the two soil zones is *fuzzy*. However, for a forest divided into separate landparcels, we can clearly say what tree belongs to what landparcel. The boundaries between the land parcels are *crisp* and thus *certain*.

In contrast, let us consider the position of a moving vehicle whose location is not exactly known, e.g., a car is in New York. This example is characterized by a *lack of knowledge* about the car’s location. The fact that the car is somewhere is precise. However, the lack of knowledge we have about its position introduces *uncertainty*. Without further knowledge, we can only give the probable area the car is in.

These examples indicate that the distinguishing element between fuzzy and non-fuzzy facts is a *crisp boundary*, i.e., when we cannot clearly say what belongs to what. The concept of boundary introduces the *interior/exterior* notion, i.e., what is within the boundary and what is outside. Spatial fuzziness occurs (a) in the relationships among spatial objects and (b) in spatial attributes.

On the other hand, the distinguishing element between uncertain and certain facts is *the lack of, or the error in our knowledge,* i.e., not sufficient knowledge about an otherwise precise fact. As a result, spatial uncertainty can refer to the degree of knowledge we have about an object’s position. Uncertainty about an object's position leads to uncertainty about the spatial relationship among this object and its neighbors, e.g., if the exact boundary of a land parcel is not known, then, the exact relationships with its neighboring land parcels are not known either. Furthermore, uncertainty can exist for spatial attributes, when knowledge about them is limited.

4.2 Indeterminate Geometry

In this section, we examine in what ways fuzziness and uncertainty affect the concept of geometry. This is essential in defining spatial objects and spatial attributes; spatial relationships are defined in terms of the geometry of spatial objects.

Points and *regions* are the most commonly met geometries in spatial applications, while *line*, is a special case of a region. Here, we only consider simple geometries, i.e., points and regions with no holes and no disconnected parts. The following cases exist: (a) Uncertain point. A point can be crisp and uncertain, e.g., we know the approximate position of a car and can give probabilities for its location. (b) Fuzzy point. This case is not applicable, since the concepts of boundary and interior/exterior do not exist here. (c) Uncertain region. Consider the example of a landparcel with "not-exactly-known" (missing data) boundaries. (d) Fuzzy region. Since a region is determined by its boundaries (something is inside/outside, or left/right), a region can be fuzzy, e.g., consider soil zones, whose boundaries are not crisp, but transitional.

Indeterminate Points. We model *Space* as a set of points, homeomorphic to $\mathbb{N}^2$. The exact position of an object with geometry point is determinate, if it can be mapped onto a single point $p \in \mathbb{N}^2$. The position is indeterminate, if it can only be mapped to a set of points, i.e., the exact position is unknown. A probability function describes the likelihood for each point to be the position, e.g., uniform distribution tells us that there is an equal chance for each point. The probability mass function, p_x, for the indeterminate point x is

$$p_x(i) = P[x = i] : i \in \{\mathbb{N} \times \mathbb{N}\} \tag{5}$$

where $P[x = i]$ is the probability that the position is mapped to point *i*, with *i* being a Cartesian coordinate. The probability that the position is outside the point set is 0. Further, all indeterminate positions are considered to be independent.

What applies to time points, can also be applied to indeterminate points in the spatial context; probability distributions describing positional indeterminacy can always be interpreted as fuzziness.

Indeterminate Regions. Indeterminate regions comprise uncertain and fuzzy regions. A *region* is a part of space bound by a connected set of points, the boundary. It can be determinate if the boundary points are determinate. Consequently, indeterminate points bound an indeterminate area. The following example illustrates this point.

Uncertain Regions. Consider a map made up of two discrete regions, A and B, sharing a common boundary. If we repeatedly digitize the map, assuming that our process introduces errors, we obtain a set of points that lies close to the actual boundary line. However, there will be more points closer to the actual location of the line than further away from it. Due to lack of knowledge, this distribution might take the form of a normal distribution whose mean is centered at the "true" location of the line. In Fig. 2a, we show the normal distribution of a particular boundary point. Fig. 2b shows the probability function in the continuous case.

Analogously, we can describe this uncertain region using a membership function. To determine this function that returns the grade to which an arbitrary point in space

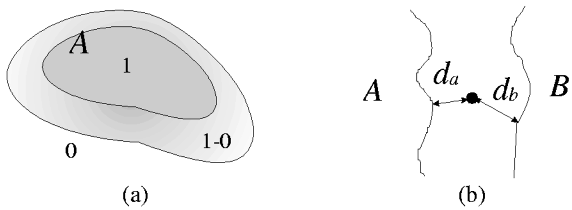


Fig. 2. Boundary point probability

belongs to an area, we split the underlying space into three parts: (a) the core of the area, (b) the boundary region, and (c) the outside. Consequently, a membership function for area *A* can be specified as follows.

$$\mu_A(i) = \begin{cases} 1 & i \in A \wedge i \notin B \\ \sum p(x) & i \in A \wedge B \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Area *B* stands for the outside of area *A* and $p(x)$ is the probability mass function of a point for being in area *A*. The argument of the membership function is a point and it returns a grade for the membership of this point in area *A*. The grade is 1 if the point is a definite member of the area and 0 if it is definitely not a member of the area. Otherwise the grade is between 1 and 0 (cf. Fig. 2a).

Fuzzy Regions. The above approach is only feasible when the probability function is known and simple, i.e., there is one probability function describing the distribution of all points in the boundary. If there were many probability functions, the membership function would become too complex to be useful. On the other hand, in some cases, we do not have "any information at all" about the boundary of a region. Consider again the transition between soil zones. The boundary exists because of the very nature of a phenomenon that is not crisp and, thus, to give a probability function describing it is not possible, or does not make sense. This illustrates the critical case for which fuzziness relieves uncertainty. We can still derive a valid membership function in assuming a smooth and steady transition from one zone to the other. A membership function for soil zones, as shown in Fig. 2b, could be characterized by the following formula (cf. [17]),

$$\mu_A(x,y) = \begin{cases} 1 & \text{if } (x,y) \in A \\ 1 - d_a / \left(d_a + d_b\right) & \text{if } (x,y) \notin A \wedge \ (x,y) \notin B \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

where d_a and d_b are the distances from a point (*x*,*y*) to the core area of the soil zones *A* and *B*. A formula for a distance *d* from an arbitrary point given by its coordinates (*x*,*y*) to an area *A* with the boundary B_A is as follows

$$d\left(\left(x,y\right), B_A\right) = \min\left\{\text{dist}\left(\left(x,y\right),\left(m,n\right)\right) \mid \left(m,n\right) \in B_A\right\} \tag{8}$$

where dist(*p*,*q*) is the Euclidean distance between two points $p,q \in \mathbb{R}^2$.

Above, the assumption is that the transition between the soil zones is linear. However, the effect of other transitions on the membership function would change the formula describing the membership grade for positions outside the core.

5 Spatiotemporal Indeterminacy

After showing the nature of spatial and temporal indeterminacy as well as the way to model it, we describe the combined phenomenon, *spatiotemporal indeterminacy*. Consider the example of a moving vehicle, it is reasonable to assume that its extent does not matter in a given application, and, thus, can be reduced to point. To record its movement, we sample the object's position. We cannot answer queries about an object's movement at times in-between position samples unless we interpolate the positions, e.g., linear interpolation.

For areal objects, the change of position includes the change of their centroid and shape, which has to be interpolated as well. Consider the indeterminate region example of an island. Tides have (a) a short-term effect on its coastline, whereas (b) over a longer period of time a general drift can be observed as well. If one is only interested in the general drift, the tidal effect can be modeled as a fuzzy boundary that changes over time.

5.1 Spatiotemporal Scenarios and Indeterminate Change

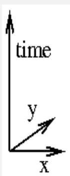
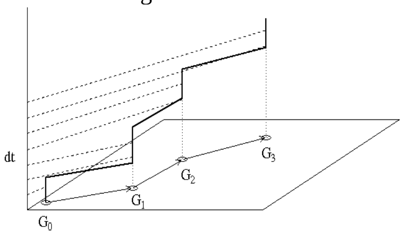
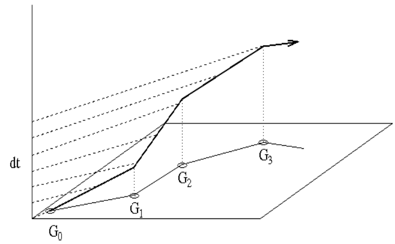
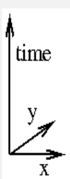
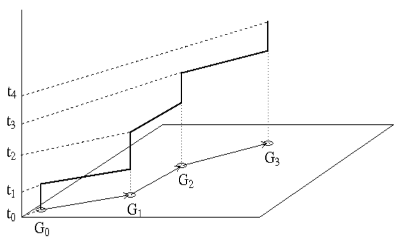
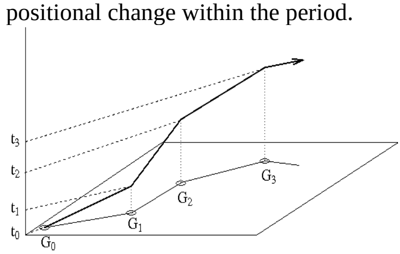
Change, or evolution, is the most important concept in the spatiotemporal context, and will in the following serve as the basis to evaluate spatiotemporal indeterminacy. As stated in literature [4], [8], [15], change (a) can either occur on a discrete or on a continuous basis and (b) can be recorded in time points or in time periods.

Table 1 illustrates the four *change* scenarios encountered in the spatiotemporal context by using a 3-dimensional representation of the temporal change of geometry. Space (*x*- and *y*-coordinates in the horizontal plane) and time (time-coordinate in the vertical direction) are combined to form a three dimensional coordinate system. In the change scenarios, the elements that can be indeterminate (with respect to an object) are *geometry*, *time point*, and *time interval*. We use a point geometry to keep the illustrations simple. However, the same change scenarios apply to other geometries. A discrete change of geometry from G_i to G_{i+1} is indicated by using an arrow in the spatial plane as opposed to a line in case of a continuous change. In the following, we examine each scenario with respect to indeterminacy.

The first case, *Scenario 1* in Table 1, is the *discrete change of a geometry recorded in time points*. Geometry stays constant for some time and then changes instantly. It is sampled at constant time intervals *dt*. The geometry and/or the time point can be indeterminate.

The second case, *Scenario 2* in Table 1, is the *continuous change of a geometry recorded in time points.* We sample a constantly changing geometry at time intervals *dt*. Knowing a geometry only at time points has two implications, (i) recording geometries at points means assessing a momentary situation without inferring anything about the geometry prior or past the time point. Consequently, (ii) time and space are

Table 1. Four spatiotemporal change scenarios

Change Time	Discrete	Continuous
Point 	**1)** Geometry is recorded at a time point. It may or may not differ from the previously recorded one. We do not know when the change occurred. 	**2)** Geometry is sampled at *time points*. In between time points we have no knowledge about the geometry. 
Period 	**3)** Geometry is valid for a given *time period*. After a change, a new time period starts. 	**4)** Geometry is sampled at time points, the starting and end points of the time period. A time period is assigned a *"change" function* that models the positional change within the period. 

independent; not knowing the exact extent of the geometry does not affect the time interval and vice versa.

In contrast, *Scenarios 3* and *4* in Table 2, suggest that a *change function* of the form $C : t_x \to G_x$ exists that determines a geometry G_x for a time point t_x in an interval spatially bound by the two geometries G_i and G_{i+1} and temporally bound by the time interval $T_i = [t_i, t_{i+1}]$. The change function *C* can be different for every time interval.

The third case, *Scenario 3* in Table 1, is the *discrete change of a geometry recorded in time intervals.* The objective is to "begin" a new interval when a spatial change occurs, i.e., new time intervals start at the time points t_0 through t_4. The geometry is constant within a time interval. Spatial and temporal indeterminacy affect each other. Dealing with indeterminate spatial extents, e.g., uncertainty induced by measurement errors, implies that the time point at which a change occurs cannot be detected precisely. On the other hand, having an indeterminate temporal event, e.g., clock errors, introduces spatial indeterminacy.

The last and most complex case, *Scenario 4* in Table 1, is the *continuous change of a geometry recorded in time intervals.* This case is based on the fact that for a

Table 2. Change scenarios without temporal indeterminacy

Geometry (G_i, G_{i+1})	**Time (t_i, t_{i+1})**	**Change**
Determinate	Determinate	$C: t_x \rightarrow G_x$, where G_x, depending on the change function, is determinate or indeterminate ($\tilde{G}_x$)
Indeterminate	Determinate	(a) $C: t_x \rightarrow \tilde{G}_x$, where $\tilde{G}_x$ represents a probability, $P_x(i)$, or a membership function, $\mu_x(i)$ (b) $\mu_x(i,t)$ or $P_x(i,t)$

given time interval $T_i = [t_i, t_{i+1}]$, there exists a change function that models the transformation from geometry G_i to G_{i+1}. Each of these factors, i.e., (i) the time interval, (ii) the geometry, and (iii) the change function, can be subject to indeterminacy.

In the simplest case, the geometry G_i and G_{i+1} and the time interval T_i are *determinate,* and the change function returns a determine geometry G_x for a given time point $t_x \in T_i$. Here, we assume that the change function returns the geometry coinciding with the actual movement. Is this not the case, the change function *interpolates* in between the geometries G_i to G_{i+1} and returns an indeterminate geometry. An example is to use linear interpolation, i.e., the two geometries G_i to G_{i+1} are considered to be the endpoints of a line. Section 5.2 gives an elaborate example of a change function for this case.

If we further allow G_i and G_{i+1} to be *indeterminate,* our change function would in any case return an indeterminate G_x. In the following, we use the “~” symbol on top of the parameter to denote indeterminacy. This means that if a geometry is described by a probability or membership function, this very function is subject to change in the time interval T_i.

Following the idea from before, we would have a change function that returns a probability or membership function for a given t_x (cf. Table 2(a)). However, by integrating the temporal component, we obtain a spatiotemporal probability or membership function, i.e., a function that changes with time (cf. Table 2(b)).

Until now, we always considered time to be determinate. We use time points to determine the start and the end of the current time interval T_i, and to denote the time point in question, t_x. In case t_i and t_{i+1} are indeterminate, we cannot state the beginning and the end of the time interval precisely. Thus, the association of a geometry (indeterminate or not) to a time point becomes indeterminate. However, this affects mainly the change function and can be considered in adapting its form. In considering an indeterminate time interval, we cannot, for any time point in the time interval, give

Table 3. Change scenarios incorporating temporal indeterminacy

Geometry (G_i, G_{i+1})	**Time (t_i, t_{i+1})**	**Change**
Determinate	Indeterminate	$C: \tilde{t}_x \rightarrow \tilde{G}_x$
Indeterminate	Indeterminate	(c) $C: \tilde{t}_x \rightarrow \tilde{G}_x$, where $\tilde{G}_x$ is either a probability, $P_x(i)$, or a membership function, $\mu_x(i)$ (d) $\mu_x(i,\tilde{t})$ or $P_x(i,\tilde{t})$

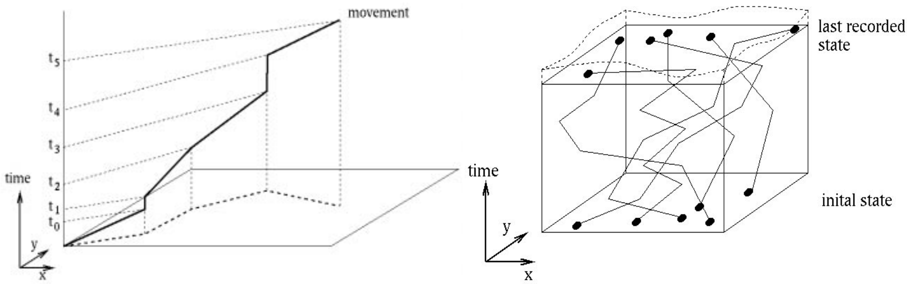


Fig. 3. Movements and space

a geometry as it would be unaffected by determinate time, but the indeterminate time contributes some additional indeterminacy. Table 3 adapts the approach shown in Table 2 to cover this case.

The central element of spatiotemporal indeterminacy is the change function manipulating geometries. This function can be seen similar to a morphing algorithm between different instances of geometries, i.e., point, line, or region. Next, we give an example illustrating the aforementioned concepts.

5.2 An Example of Use – Tracking Vehicles

Consider the application scenario in which we track the continuous movement of taxis equipped with GPS devices that transmit their positions to a central computer using either radio communication links or cellular phones.

Acquiring Movement – Sampling Moving Objects. To record the movement of an object, we would have to know the position on a continuous basis. However, practically we can only sample an object's position, i.e., obtaining the position at discrete instances of time such as every few seconds.

The solid line in Fig. 3a represents the movement of a point object. Space (x- and y-axes) and time (t-axis) are combined to form one coordinate system. The dashed line shows the projection of the movement onto two-dimensional space (x and y coordinates). A first approach to represent the movements of objects would be to store the position samples and interpolate the in-between positions. The simplest approach is to use linear interpolation. The sampled positions become the end points of line segments of polylines. The movement of an object is represented by an entire polyline in three-dimensional space. In geometrical terms, the movement of an object is termed a *trajectory* (we will use "movement" and "trajectory" interchangeably). Fig. 3b shows a spatiotemporal space (the cube in solid lines) and several trajectories (the solid lines). The top of the cube represents the time of the most recent position sample. The wavy-dotted lines symbolize the growth of the cube with time.

Measurement Error. An error can be introduced by inaccurate measurements. Using GPS measurements in sampling, the error can be described by a probability function, in our case, a bivariate normal distribution P_1.

$$P_1(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \tag{9}$$

where σ is the standard deviation. For details on this error measure refer to [13].

Which Scenario? In Table 1 of Sect. 5.1, the sampling approach to assess the movement of objects is characterized by scenario 4. Tables 2 and 3 establish a foundation for giving a change function in between sampled position. Table 3 gives function templates in case the times of sampling are not known precisely. However, GPS allows for precise timing and, thus, we neglect the effects of time. In Table 2, Scenario 1 (determinate geometry) gives a function template in case the sampled positions are known precisely. GPS measurements are accurate but not precise. Scenario 2 (indeterminate geometry) seems to be a match for our problem. Next we show how to establish a change function to determine the position of the moving object in-between sampling. We initially assume precise position samples.

Sampling Uncertainty. Capturing the position using a GPS receiver at regular time intervals introduces *uncertainty about the position* of the object for the in-between the measurements. In this section, we give a model for the uncertainty introduced by the sampling, based on the sampling rate and the maximum speed of the object.

The uncertainty of the representation of an object's movement is affected by the *sampling rate*. This, in turn, may be set by considering the speed of the object and the desired maximum distance between consecutive samples. Let us consider the example of recording taxi movements. As a requirement, the distance between two consecutive samples should be maximally 10*m*. Given the *maximum speed* of a taxi as 150*km*/*h*, we would need to sample the position at least 4.2 times per second. If a taxi moves slower than its maximum speed, the distance between samples is less than 10*m*.

Since we did not have positional measures for the in-between position samples (cf. Fig. 4a, the object could be anywhere in between position samples), the best is to *limit the possibilities of where the moving object could have been*. Considering the trajectory in a time interval $[t_1, t_2]$, delimited by consecutive samples, we know two positions, P_1 and P_2, as well as the object's maximum speed, v_m (cf. Fig. 4b). If the object moves at maximum speed v_m from P_1 and its trajectory is a straight line, its position at time t_x will be on a circle of radius $r_1 = v_m(t_1 + t_x)$ around P_1 (the smaller dotted circle in Fig. 4b). Thus, the points on the circle represent the maximum distance of the object from P_1 at time t_x. If the object's speed is lower than v_m, or its trajectory is not a straight line, the object's position at time t_x will be somewhere within the area bound by the circle of radius r_1.

Similar assumptions can be made on the position of the moving object with respect to P_2 and t_2 to obtain a second circle of radius r_2. The constraints on the position of the moving object mean that the object can be anywhere within the intersection of the two circular areas at time t_x. This intersection is shown by the shaded area in Fig. 4b. We use the term *lens* for this area of intersection. We assume a uniform distribution

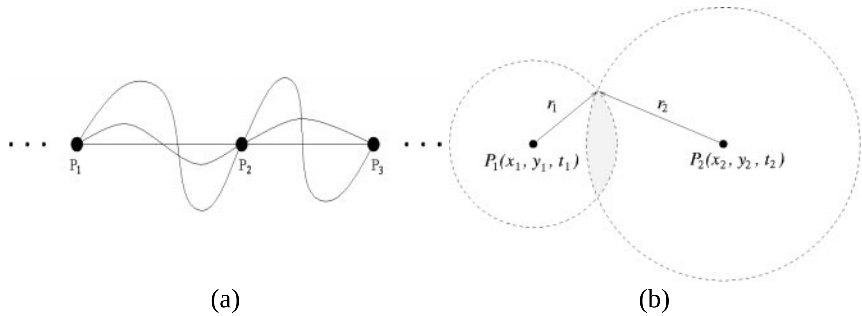


Fig. 4. (a) Possible trajectories of a moving object, (b) uncertainty between samples

for the position within the lens, i.e., the object is equally likely anywhere within this lens shape.

The sampling error at time t_x for a particular position can be described by the probability function of Equation 10, where r_1 and r_2 are the two radii described above, s is the distance between the measured positions P_1 and P_2, and A denotes the area of the intersection of the two circles.

$$P_2(x,y) = \begin{cases} 1/A & \text{for } x^2 + y^2 \le r_1^2 \wedge (x-s)^2 + y^2 \le r_2^2 \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

To eliminate the radii in favor of the max speed and times, we can substitute $v_m(t_1 + t_x)$ and $v_m(t_2 - t_x)$ for the r_1 and r_2, respectively. This function describes the position of the moving object in between position samples. Thus, this function is an instance of the function template as described in Scenario 1 of Table 2.

Combining Error Sources – a Global Change Function. Table 2 gives a template of a change function that incorporates indeterminate positions. Using our example, this translates to adapting Equation 10 such that the values for x and y are not precise but affected by the measurement error. A mathematical framework suitable for this problem is *Kalman filtering* [11], which combines various error prone measurements about the same fact into a single measurement resulting in a smaller error. This mathematical framework stipulates a method to combine uncertainty to reduce the overall error. Examples of applying Kalman filtering to the domain of vehicle navigation are the integration of three independent positioning systems such as dead reckoning, map matching, and GPS, to determine the precise position of vehicles [12].

6 Conclusions and Future Work

The work presented in this paper concerns the spatial, temporal, and spatiotemporal indeterminacy, i.e., fuzzy and uncertain phenomena. We first show how the fundamental modeling concepts of *spatial objects, attributes, relationships, time points, time periods*, and *events* are influenced by *indeterminacy*. Next, we focus on the change of spatial objects and their geometry in time. We argue that change can occur on a discrete and on a continuous basis, as well as it can be recorded in time points and time periods. By combining these concepts, we present four different change scenarios, which are affected by indeterminacy to a various degree. The indeterminacy of change is formalized and combines the spatial and temporal concepts. Finally, the rather general concepts are applied to existing application areas. We discuss uncertainty existing in the context of moving-point-object applications. We give a change function to describe the position of moving objects in time, based on positional samples. The change function is influenced by measurement errors and sampling uncertainty.

Although mentioned, the paper does not discuss, directly, indeterminacy as related to relationships among spatial, temporal, or spatiotemporal objects. An extension of this work towards this direction is essential. Also, the mathematical models we presented are concrete enough to describe and motivate indeterminacy related to the temporal, spatial, and spatiotemporal domain. However, to actually implement these concepts, more detailed mathematical formulas are needed. Finally, in a more general framework, this work points towards the development of spatiotemporal data types and data structures incorporating indeterminacy.

References

1. Burrough, P.A., MacMillan, R.A., amd van Deursen, W.: Fuzzy Classification Methods For Determining Land Suitability from Soil Profile Observations and Topography. *Journal of Soil Science*, 43, pp. 193-210, 1992.
2. Cheng, T. and Molenaar, M.: Diachronic Analysis of Fuzzy Objects. *GeoInformatica* 3(4), pp. 337 - 355, 1999
3. Chrisman, N.: A Theory of Cartographic Error and Its Measurement in Digital Databases. In *Proceedings Auto-Carto 5*, pp. 159-168, 1982.
4. Claramunt, C., and Theriault, M.: Managing Time in GIS: An Event-Oriented Approach. *Recent Advances in Temporal Databases*, Springer-Verlag, pp. 142-161, 1995.
5. Dyreson, C.E., Soo, M.D., and Snodgrass, R.T.: The Data Model for Time. *The TSQL2 Temporal Query Language*, Kluwer Academics, pp. 97-101, 1995.
6. Fisher, P.: Boolean and Fuzzy Regions. *Geographic Objects with Indeterminate Boundaries*, Taylor & Francis, pp. 87-94, 1996.
7. Goodchild, M. and Gopal, S. (Eds): *Accuracy of Spatial Databases.* Taylor & Franics, 1989.
8. Güting, R., Böhlen, M., Erwig, M., Jensen, C.S., Lorentzos, N., Schneider, M., and Vazirgiannis, M.: A Foundation for Representing and Querying Moving Objects. *ACM Transactions on Database Systems* 25(1), pp. 1-42, 2000.
9. Hadzilacos, T.: On Layer-Based Systems for Undetermined Boundaries. *Geographic Objects with Indeterminate Boundaries*, Taylor & Francis, pp. 237-256, 1996.
10. Hadzilacos, T., and Tryfona, N.: A Model for Expressing Spatiotemporal Integrity Constraints. In *Proc. of the International Conference on Theories and Methods of Spatio-Temporal Reasoning*, pp. 252 - 268, 1992.
11. Kalman, R.E.: A New Approach to Linear Filtering and Prediction Problems. *Transactions of the ASME–Journal of Basic Engineering*, pp. 35-45, 1960.

12. Krakiwsky, E.J., Harris, C. B., and Wong, R.: A Kalman Filter for Integrating Dead Reckoning, Map Matching, and GPS Positioning. In *Proc. of the IEEE Position Location and Navigation Symposium*, pp. 39-46, 1988.
13. Pfoser, D. and Jensen, C.S.: Capturing the Uncertainty of Moving-Object Representations. In *Proc. of the 6th International Symposium on the Advances in Spatial Databases*, pp. 111-132, 1999.
14. Pfoser, D. and Tryfona, N.: Requirements, Definitions, and Notations for Spatiotemporal Application Environments. In *Proc. of the 6th ACM Symposium on Geographic Information Systems*, pp. 124-130, 1998.
15. Pfoser, D. and Tryfona, N.: Capturing Fuzziness and Uncertainty of Spatiotemporal Objects. *TimeCenter Technical Report*, 2001.
16. Pfoser, D., Jensen, C., and Theodoridis, Y.: Novel Approaches in Query Processing for Moving Objects Data. In *Proc. of the 27th Conference on Very Large Databases*, pp. 395-406, 2000.
17. Schneider, M., Metric Operations on Fuzzy Spatial Objects in Databases, In *Proc. of the 8th ACM Symposium on Geographic Information Systems,* 2000.
18. Shibasaki, R.: Handling Spatiotemporal Uncertainties of Geo-Objects for Dynamic Update of GIS Databases from Multi-Source Data. In *Advanced Geographic Data Modeling*, Netherlands Geodetic Commission, Publications on Geodesy, 40, pp. 228-243, 1994.
19. Vazirgiannis, M.: Uncertainty Handling in Spatial Relationships. In *Proc. of the ACM Symp. on Applied Computing*, 2000.
20. Worboys, M.: Imprecision in Finite Resolution Spatial Data. *GeoInformatica* 2(3), pp. 257-279, 1998.